



Numeracy, Numerical Cognition, Ontology, Cognitive Acceleration, Cultural-Historical Activity Theory (CHAT)

Topic:

The fundamental nature and existence of numbers and mathematical objects.

The use of Computational Ontology (structured knowledge representation) to study how we understand and use numbers.

Precepts:

All is ONE $\mathbb{Q}$

All rational numbers are an expression of a **single entity**. This touches on the most fundamental concept of abstract algebra and number theory, specifically the properties of the rational number field ($\mathbb{Q}$) and the definition of a **Unit Element**.

Rational Numbers as Multiples of the Unit Element (1)

All rational numbers can be fundamentally understood in relation to the **multiplicative identity**, which is the **number 1**.

- Every rational number q can be expressed as the *product* of the unit element (1) and q itself ($q \times 1 = q$).
- More deeply, within the structure of rational numbers, every number is a result of basic operations (addition, subtraction, multiplication, division) starting only with the number 1.

The Role of the Number 1

1 acts as the **generator** for the system under multiplication. The entire set of rational numbers is defined as the smallest field containing the integers, which are in turn generated by repeated addition and subtraction of 1.

The fact that any non-zero rational number p/q has a multiplicative inverse (q/p) means that the "size" of the unit can be **scaled and subdivided infinitely**, but every resulting number always maintains that essential relationship back to the foundational "whole" or "unit" quantity: 1.

While all rationals stem from the single entity of the number 1 through arithmetic operations, they are not all *equal* to one entity (e.g., $1/2 \neq 1$). The set of all rational numbers represents a **field** that is *generated* by the single unit element, 1.

Bounding The Infinite ∪

There are more *real numbers* **between** 0 -1 than there are *natural numbers*, *integers*, and *rational numbers* combined.

All **subtraction** is secretly **addition** of the **negative**

All **division** is **multiplication** by the **reciprocal***

All **multiplication** is repeated **addition**

All additions are iterations of the **Unit Element**

Divinity of 0:

Bantu-Congo (Central Africa): Nzambi

Northeast Africa (Egyptian): Nefer

Standard Model: λ

1. 0 is Independent, 0 is a preexisting constant. It doesn't need a function to create it, and it doesn't need a predecessor to justify its existence. It is simply "there" by axiom. Every other number must "ask" the successor function to exist, but 0 exists by decree.

2. All other numbers are "Derived". Every other natural number is essentially just "0 with extra steps."

- 1 is just 0 with one (S) applied: (S(0)).
- 2 is just 0 with two (S) applied: (S(S(0))).
- 1,000,000 is just 0 with a million (S) applied. If you took 0 away, the entire infinite chain of numbers would vanish because they have no "anchor" to start from.

The Four Basic Operations ⊕

Bantu Kongo Cosmology: Dikenga Cross

Standard Model: 4 fundamental interactions

The rational numbers are the numbers that can be constructed through a finite sequence of arithmetic operations ($\times$, $\div$, $+$, $-$) on integers (or really, just from the number 1). Any rational number has some arithmetic sequence that constructs it. Anything that can't be constructed with such a finite sequence, is defined as an irrational number. In other words, irrational numbers are those whose arithmetic construction (if it exists) must be infinite.

Division

Division is the act of viewing the **dividend** (numerator) as the base **unit element** of a new ratio.

Viewing the numerator as the base unit element (1) is a way of defining a Rational Number (a fraction) as the relationship between the numerator and denominator.

All **division** result in a **quotient**.

All **quotients** represent a **ratio**.

All **ratios** are representations of a **unit entity** (dividend/numerator) in terms of divisor/denominator.

All **remainders** of division will be less than the **divisor**.

Common Denominator

To subtract fractions, we create a Lowest Common Denominator—conjuring **one** in a form that normalizes both fractions:

$$1/3 - 1/4 = (1/3 \cdot 4/4) - (1/4 \cdot 3/3) = 4/12 - 3/12 = 1/12$$

Balance Through Opposition

Solving equations means applying **opposite operations** to **both sides**, preserving equality while isolating the unknown:

$$x + 5 = 12$$

$$x + 5 - 5 = 12 - 5$$

$$x = 7$$

Transformation: (Equivalence-Preservation)

Most mathematics does not consist of calculation, but **equivalence-preserving operations** grounded in the axiology of unity and infinity.

The Unit Element: Infinite Transformations

The number 1 can be reshaped into infinite equivalent forms without changing its value:

$$1 = \sqrt{3}/\sqrt{3} = e^0 = 2/2 = \cos^2\theta + \sin^2\theta = 100\%$$

Example: To rationalize $12/\sqrt{3}$, we *transmute* 1 into $\sqrt{3}/\sqrt{3}$:

$$12/\sqrt{3} \times \sqrt{3}/\sqrt{3} = 12\sqrt{3}/3 = 4\sqrt{3}$$

Example: To simplify $\frac{12}{\sqrt{3}}$, we multiply by One in the form of $\frac{\sqrt{3}}{\sqrt{3}}$.

$$\frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

We didn't change the value; we multiplied by a form of 1 to **rationalize the denominator**.

OSCI

Mathematics is not rigid—it is **fluid transformation**. Every number, every expression exists in **infinite equivalent forms**. The mathematician's art is choosing which form reveals the solution.

Structuring The Infinite

Now that we know we are dealing with one substance we now have the power to transform that substance to study it. Each branch of mathematics studies **the one** in specific contexts (e.g. arithmetic, algebra, geometric algebra, calculus etc.) and these context shape it's form. Each branch of math has its own unique grammar and syntax.

Mathematical Notation: Structured Knowledge Representation

Yoruba Ifá: Patakis
Mathematical Logic: Peano Axioms

Mathematical notation changes depending on the field of study. Different branches of math highlight different aspects of reality, requiring notation that can measure, shape and express those aspects in a specific context. For example although many symbols may be shared between geometric algebra and statistics the syntax structure is different for each branch.

Mathematics is not a universal symbol language; it is a stack of local grammars. The specific symbols used are **arbitrary**. There is no "natural" law stating that two crossed lines (+) must mean addition. The symbol meanings and mathematic associations were culturally developed then adopted through convention over time. Mathematics is often called a "universal language," but its **notational script** is a masterpiece of **cultural evolution**, trial and error, and eventual global adoption.

Symbolic Meaning Across Multiple Mathematical Domains

Identical mathematical symbols often carry divergent meanings across various branches of mathematics.
Notation in each branch has its own unique syntax.

Note how the same symbol migrates from *quantity* → *relation* → *space* → *process*

Symbol	Arithmetic	Algebra	Geometric Algebra	Calculus
• (Dot)	Multiplication (x)	Multiplication	Inner product (projection → scalar)	Time derivative ($\dot{x}$): rate of change
— Fraction Bar (Plus dots)	Noun: % ÷ part of a whole (3/4)	Verb: division / ratio (x/y)	—	Ligature: derivative (dy/dx), slope
² Superscript	Repeated multiplication	Power operation	Norm collapse: magnitude ² → scalar	Second derivative: curvature
() Parentheses	Order of operations	Grouping / binding	—	Process control (e.g., $f^{(2)}$, evaluation order)
0 (Zero)	Nothing / null quantity	Additive identity	Origin (reference point)	Boundary / transition state
' Prime	Primality marker	—	Related / transformed element	Derivative (rate of change)
+ Plus	Verb: Addition	Formal sum	Vector addition (translation)	Accumulation of change
∫ Integral	—	—	—	Accumulation / totality (operator on change)

Arithmetic: Context-Bound Pattern Recognition

Ontology: Discrete quantities in everyday experience

Grammar:

- Juxtaposition = addition (12 2/3)
- Explicit symbols for operations ($\times$, $\div$, $+$, $-$)

Arithmetic emerged from practical counting and measurement. The notation reflects **concrete world-sensing** where numbers represent tangible things. Mixed numbers (12 2/3) evolved from fractions representing real divisions (12 whole pies plus 2/3 of another pie).

Algebra: “Putting back together”

Algebra involves the use of symbols (letters and numbers) to resolve an unknown value.

Ontology: Relationships between unknown quantities, functional dependencies

System	Type	Symbol Form	Example	Value/Function
Egyptian Hieroglyphs	Triliteral	single sign	 pr	3 consonants
	Biliteral	single sign	 pr	2 consonants
	Uniliteral	single sign	 r	1 consonant
Algebra	Literal - Constant	letter/symbol	a, b, π , φ	fixed
	Literal - Variable	letter	x, y	variable
	Expression	combination	$3x + 2y$	variable + constant

Grammar:

- Juxtaposition = multiplication ($3x$, $(5/8)x$)
- Variables represent varying quantities
- Equations express relationships

Algebra shifts from "**what is this quantity?**" to "**how do quantities relate?**" The grammar must encode:

- Parametric variation (x can be anything)
- Functional relationships (y depends on x)
- Abstract patterns independent of specific values

The notation becomes **decontextualized** (like alphabetic scripts) to enable formal manipulation without constant reference to concrete meaning.

Geometric Algebra: Spatial Structure and Orientation



Ontology: Directed magnitudes, rotations, reflections, multidimensional relationships

Product Type	Resultant Entity	Common Notation
Inner/Dot	Scalar	$a \cdot b$
Cross	Vector (Orthogonal)	$a \otimes b$
Outer	Matrix (Tensor)	$a \times b$
Wedge	Bivector (Plane)	$a \wedge b$

Bivector Relationship to Cross Product: In 3D, the wedge product is the mathematical "sibling" to the cross product. While the cross product represents the plane as a perpendicular vector, the wedge product represents the **oriented plane itself**.

Grammar:

- Juxtaposition = geometric product (ab contains multiple grades)
- $a \cdot b$ = inner/dot product (scalar result)
- $a \wedge b$ = outer/wedge product (bivector/trivector result)
- Order matters: $ab \neq ba$ (non-commutative)

Ex. The sandwich product $T = a^{-1}Sa$ reveals this perfectly. Geometric algebra needs to represent:

- **Orientation** (clockwise vs counterclockwise)
- **Dimensionality** (scalars, vectors, bivectors, trivectors)
- **Transformations** (rotations, reflections as products)

The grammar must preserve **geometric meaning**—where things are, how they're oriented, how they transform. This requires grade preservation, non-commutative operations, and distinction between different product types.

Calculus: Continuous Change and Limiting Processes

Ontology: Infinitesimal change, rates of change, accumulation

Grammar:

- dy/dx (not "d times y divided by d times x")
- $\int$ and d as inseparable operator pairs
- Limits written as $\lim \rightarrow$

Calculus represents **processes rather than states**. The notation must express:

- Infinitesimal quantities (dx , dy)

- Limiting operations (approaching but not equal)
- Integration and differentiation as inverse operations